Department of Economic & Social Affairs

DESA Working Paper No. 150
ST/ESA/2017/DWP/150

August 2017

Economic Diversification: Explaining the pattern of diversification in the global economy and its implications for fostering diversification in poorer countries

Author: Clovis Freire[1]

ABSTRACT

Economic diversification is very relevant for poorer developing countries to create jobs and foster economic development. That need has been recognized in key internationally agreed development goals. The empirical economic literature has identified several stylized facts about the pattern of diversification of economies, but the development of explanations for those patterns in general has been only loosely associated with economic theory on growth, trade, technology change and structural transformation. Making that connection is relevant because it could inform policymakers in developing countries in designing and implementing policies for promoting diversification. This paper presents a model of structural economic dynamics and endogenous technological change that is able to replicate empirical regularities related to economic diversification. The model is used to study strategies to foster diversification in poorer countries, which could help to better target action in the implementation of internationally agreed goals related to the economic diversification of these countries.

Keywords: Diversification, Economic Complexity, Structural Transformation, Productive Capacities, Economic Development.

1 United Nations, Division for Sustainable Development, 405 E. 42nd Street, New York 10017 NY, USA. freire@un.org

CONTENTS

1. Introduction. 1

2. Empirical regularities and theoretical explanations 3

3. Overview of the model 4

4. Description of the model 6

5. Replication of the stylized facts 16

6. Catch-up strategies of poorer countries. 19

7. Conclusions 24

References. 25

Appendix A. Variables of the model. 28

Appendix B. Sensitivity analysis 28

Appendix C. Product complexity 30

Acknowledgements: *I am grateful to Bart Verspagen and Bart Los for their advice in the development of the model presented in this paper. I thank two anonymous reviewers for their comments on the paper. I am also grateful to David Le Blanc for his comments on an earlier version of the paper. All errors are my own.*

Typesetter: *Nancy Settecasi*

UNITED NATIONS
Department of Economic and Social Affairs
UN Secretariat, 405 East 42nd Street
New York, N.Y. 10017, USA
e-mail: undesa@un.org
https://www.un.org/development/desa/publications/working-paper

Economic Diversification: Explaining the pattern of diversification in the global economy and its implications for fostering diversification in poorer countries

1 Introduction

Economic diversification is very relevant for developing countries, and in particular the least developed, landlocked developing and small island countries, to create jobs and foster structural transformation and economic development. That need has been recognized in key internationally agreed development goals including the 2030 Agenda for Sustainable Development, the Programme of Action for the Least Developed Countries for the Decade 2011-2020 (Istanbul Programme of Action), the Vienna Programme of Action for Landlocked Developing Countries for the Decade 2014-2024, and the Small Island Developing States Accelerated Modalities of Action (Samoa Pathway).

Much of the scientific results that have informed the political push towards economic diversification are a product of the literature concerned with economic complexity. That stream of literature has uncovered stylized facts about the pattern of diversification of economies, but the development of explanations for those patterns in general has been only loosely associated with the economic theory on growth, trade, technology change and structural transformation.

There is, therefore, an opportunity for exploring ways to link these literatures to expand our knowledge about the relationship between diversification and economic development. A better understanding of this relation could be of great interest to policymakers in developing countries in designing and implementing policies and strategies for promoting inclusive growth and catch up with developed economies. It could help to answer questions such as: What is the relationship between economic diversification, structural change and economic growth? How does the level of economic diversification affect the number of possible opportunities for future diversification? Which strategies could governments in developing countries follow to facilitate the emergence of more productive economic activities?

This paper makes a contribution in that direction. It presents a model of endogenous technological change that is rooted in a theoretical framework of structural economic dynamics and that at same time is able to replicate empirical regularities related to economic diversification.

The model is used in computer simulation experiments to study the effect of different strategies to promote diversification of poorer countries. The results presented in this paper suggest that an effective strategy is to focus on the emulation of production that already exist in more diversified countries, and target products that require a similar set of technologies in existence in the country, but that have higher complexity than the average of the country's exports.[1]

The paper is structured as follows. The next section presents some of the empirical regularities uncovered by the literature on economic complexity and reviews some of the models that have been proposed to explain those stylized facts. Section three presents an overview of the model, and its formal description is presented in Section four. Section five presents the results of computer simulations that show how the model replicates the stylized facts discussed in section two. Section six presents a study of catch-up strategies of poorer countries using the model. Section seven concludes.

1 A product is considered more complex than another when it is produced by fewer countries that are also more diversified, which suggests that it requires a more exclusive set of technologies to be produced.

2 Empirical regularities and theoretical explanations

This section discusses theoretical explanations of three stylized facts associated with diversification: (1) a positive association between the export diversification of a country and its total GDP, (2) a negative association between diversification and the average number of countries that can export a similar basket of products; and (3) the path dependence in the diversification process.

First, empirical evidence shows that more diversified countries are associated with higher levels of GDP (ESCAP, 2011, 2014, 2015; Lei and Zhang, 2014). Figure 1.A illustrates this association by showing the country's total GDP in current terms along the horizontal axis and the country's diversification in number of products for the year 2013 along the vertical axis.[2]

Second, as economies diversify they tend to export products that are slightly less ubiquitous than their existing exports (e.g. Hausmann and Hidalgo 2011; ESCAP, 2011 and 2015). Ubiquity is defined as the number of countries that export a product. That empirical regularity is illustrated in Figure 1.B, in which the horizontal axis shows the diversification in terms of number of products, and the vertical axis shows the average ubiquity of the products exported by a given economy.

The third empirical regularity is that the existing product-mix of a country affects the potential new products that could emerge in the economy. Diversification, therefore, is path dependent. That empirical regularity is highlighted in different strands of the literature (e.g. Gerschenkron, 1962; Dosi, 1982, 1988; Hidalgo, Klinger, Barabasi and Hausmann, 2007). A result of the path dependency is that it seems difficult for countries to "leapfrog", moving directly from the production of one product to another that is far away in terms of productive capacities.

These and other empirical results related to economic diversification, which have been highlighted in the literature of economic complexity (e.g. Boschma, 2005; Hidalgo et al., 2007; Arthur, 2009; Hausmann and Hidalgo, 2011; Balland, Boschma and Frenken, 2015), have attracted attention to the need to put forward new hypotheses to explain and understand how they are generated.

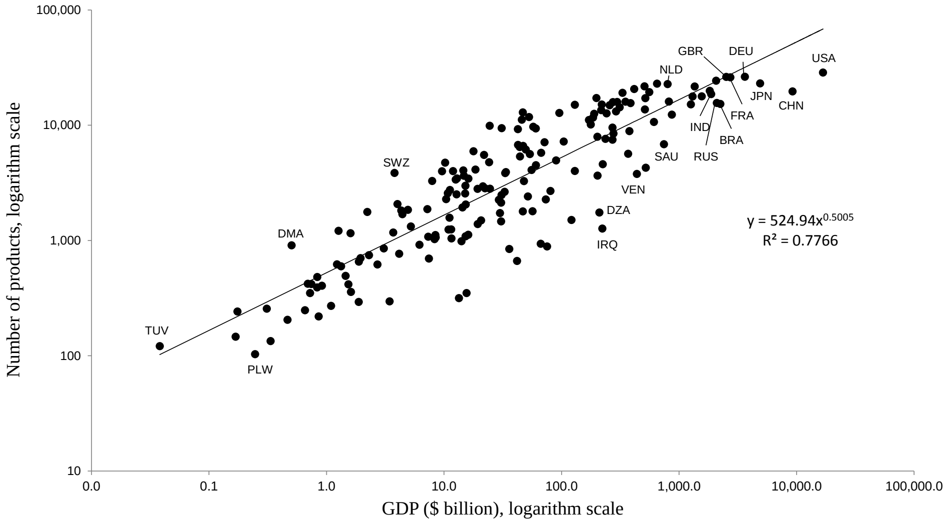
New hypotheses are needed because economic theories have not particularly focused on economic diversification. Those theories that focus on the optimum allocation of scarce goods have no place for economic diversification. The trade literature is an example of that exchange paradigm. Even the new trade theory (e.g. Krugman, 1979), which assumes that products come in different varieties that are imperfect substitutes, does not predict which country will specialize in which product because it assumes a continuum of symmetric products.

Economic theories that focus on technological progress may be able to study the process of diversification. For example, the literature on growth theory has emphasized the key role of technological change. Within that strand of literature, aggregated models (e.g. Solow, 1956, 1957) by design in general do not deal with diversification. Some endogenous growth models have considered expanding variety as the driver of growth (e.g Romer, 1990; Grossman and Helpman, 1991). However, they formulate a continuum of goods that have no intrinsic difference from each other, and therefore do not address the characteristic path dependence in the diversification process.

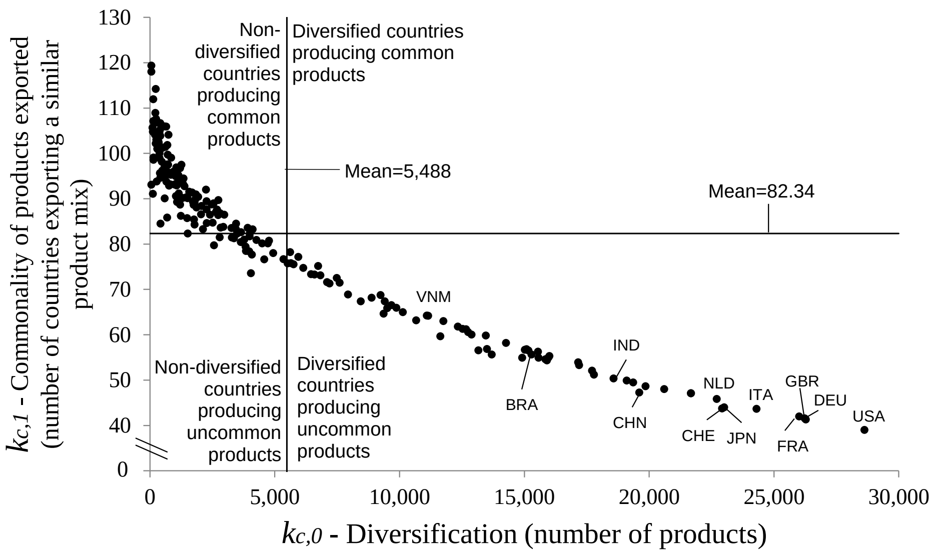
2 Other studies focusing on the association between diversification and income per capita have also found that diversification is associated with economic development for most of the development trajectory of a country (Imbs and Wacziarg, 2003; Cadot, Carrere and Strauss-Kahn, 2012). That relationship is shown to be non-monotonic, following an inverted U-shaped curve. This paper highlights the less studied relationship between diversification and total GDP.

Figure 1
Selected stylized facts related to diversification

A) Higher output is associated with diversification, 2013



B) As economies diversify, they produce more exclusive products, 2013



Source: Author's calculations based on data from UN Comtrade and from the World Bank's World Development Indicators (WDI).

Notes: Number of products exported (diversification) is the number of category of products exported classified using HS 2002 trade data disaggregated at 6-digit level and further disaggregated by unit price as per the methodology described in Freire (2011); labels indicate countries using ISO 3-digit Alpha country code.

Structuralist growth models, on the other hand, disaggregate the analysis into sectors, which provides a framework to study diversification, but diversification is not a central element (see for example the compilation of models in Gibson, 2010 and in Setterfield, 2010). In some cases, the importance of diversification is emphasized, but no formal treatment for it is given in the model (e.g. Pasinetti's, 1993), or the model concentrates on countries in autarky and there is no path dependence in the process of diversification (e.g. Saviotti and Pyka, 2004a, 2004b, 2004c).

Therefore, new hypotheses are needed to explain those stylized facts regarding economic diversification. For example, the model presented in Hausmann and Hidalgo (2011), called binomial model, provides an explanation for stylized fact 2 (negative association between diversification of countries and the average ubiquity of their exports). The model is based on the assumptions that products require the combination of capabilities to be produced, countries only produce the goods for which they have the capabilities to produce, and they produce all the products for which they have the required capabilities. However, the model was not designed to address the stylized fact 1 and it is also not suitable to address the stylized fact 3 because it is static in nature.

Lei and Zhang (2014) propose a revision to Hausmann and Hidalgo (2011) model to replicate an empirical regularity related to stylized fact 1 of the relationship between total GDP and diversification. However, their model is also static and not able to replicate stylized fact 3.

Klimek, Hausmann and Thurner (2012) explore the dynamics of the diversification process and assume that products require capabilities to be produced and new products emerge through the combination of existing capabilities in a given economy. This model also accounts for the fact that new products may replace existing products, setting a model of Shumpeterian creative-destruction. The model does not replicate stylized facts 1 and 2 but it could be used to replicate the stylized fact 3 related to path dependency, which is ruled by the combination of existing capabilities.

Saracco, Di Clemente, Gabirelli and Pietronero (2015) also propose a dynamic model in which new products emerge as the combination of previous products. The model assumes that countries compete to obtain the ability to produce and export new products and that the potential new products that a country can produce are part of the "adjacent possible" as per Kauffman (2008).[3] The authors found that the model replicates the negative association between diversification and ubiquity of production (stylized fact 2). The model also assumes the path dependency observed in the data (stylized fact 3).

However, none of the models discussed above provides any information about economic magnitudes, such as output, growth, and employment, and none is suitable to explore policy-related questions that link diversification with the structural economic dynamics of countries.

The next section presents a description of a model that replicates all three stylized facts related to diversification and that is based on the theoretical framework of structural economic dynamics.

3 Overview of the model

To better understand the stylized facts listed in the previous section, it is necessary to have a model that considers: a) the existence of multiple sectors in an economy, and analyses how that structure changes and affects the macro-economic magnitudes (e.g. employment, GDP, total consumption and balance of trade); and b) the existence of multiple countries trading with each other.

The model proposed in this paper is based on the framework of structural economic dynamics (Pasinetti, 1993; Duchin, 2005) with endogenous technological change (Andersen, 2001), and change in consumption patterns according to a generalized version of Engel's law (Pasinetti, 1993). The model considers the impact of new products in the demand of existing production (Gualerzi, 2012), adopts Keynes-Kalecki principle of effective demand (Clower, 1965), hence the model does not assume full employment, and uses the concept of adjacent possible (Kauffman, 2008) to formalize path dependency in the process of diversification.

3 An adjacent possible can be understood as a set of all possible new products that could be created in a single step based on the combination of technologies that already exist in the economy. Using an example to illustrate, suppose country *A* produces three products, in which the first is characterized by technology *a*, the second by technology *b* and the third by technology a*b*. The adjacent possible of country *A* is a set of six products, each one characterized by one of the following technologies: *aa*, *aab, bb, ba, bab,* and *abab*.

In the model, the world is composed by a number of countries, each producing many products and trading with each other. As in Pasinetti (1993), labour is the only factor of production. The unit of the analysis of the model are the sectors that constitute an economy. A critical element of the model is that it formalizes the diversification of economies as endogenous to the model. Therefore this is a multi-country multi-sector model of a pure labour economy with economic diversification.

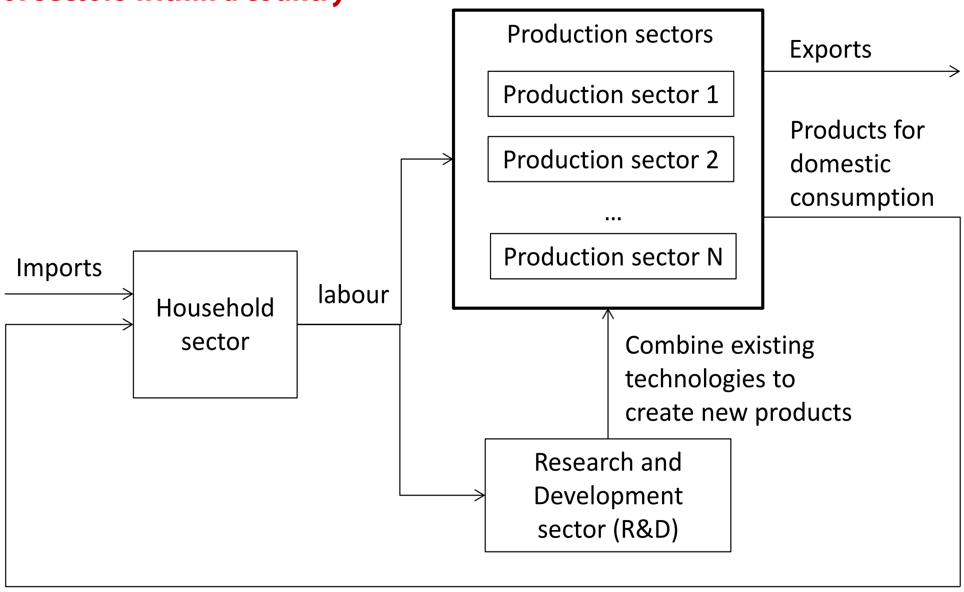
Each economy is composed by one household sector, many production sectors, and one research and development sector (R&D) (Figure 2). In each country, the household sector provides labour to other sectors and consumes products, some of them produced domestically and some abroad. Each production sector produces a single type of product, which could be consumed domestically and/or exported. Each type of commodity is recognizable by their country of origin through a brand or another product characteristic (e.g. Krugman, 1979). In each sector there is a markup price for the products. A production sector is defined by the set of labour-embodied technologies that are used in the production process.

At each unit of time and in each country, a proportion of the population is engaged in the production sectors. The share of the population that is not engaged in the production is either engaged in the research and development process (R&D) or is unemployed. The unit of labour is remunerated by a wage rate. Both wages and employment levels are endogenous in the model.

The R&D sector creates processes and products that are new to the world or to the country, which gives rise to more productive or new production sectors when they discover a new useful combination of the technologies that already exist in the country's economy.

In the short-run, the model determines which country specializes in which products based on the demand, prices of products and the amount of labour available for production. In addition to the markup mechanism, which results in more than one country selling products for the same price, the model also adopts a linear programming approach inspired by the World Trade Model proposed by Duchin (2005). The model adopts the traditional economic assumption that people prefer to pay the lowest possible price for the products that they consume. This preference for minimizing expenditure drives production towards the countries that sell at the lowest prices. The model accounts for incomplete specialization of production and trade, due to the limit in labour available for production. Therefore, the model does not assume *ex-ante* full specialization and also allows for situations in which similar products with different labour costs coexist in the global market.

Figure 2
Diagram of sectors within a country



In the long-run the economy changes with changes of consumption patterns and technical progress, both of which are endogenous in the model. We model technical progress through process and product innovation and emulation following mechanisms similar to those adopted in Andersen (2001). Both process and product innovations, which create a set of technologies that is new to the world, are assumed to be less frequent events than emulation, which is an innovation that creates a product or a process that is new only to the country.

The next section presents the formal description of the model.

4 Description of the model

The model in the short-run determines which country produces which product. In the long-run the model determines the economic dynamics over time due to trade, changes in consumption patterns and technical progress. For simplification, we assume that population sizes remain constant.

4.1 Short-run

The model assumes international trade between R countries, which together produce m different types of commodities. Each country's economy is composed by a household sector, m production sectors and one R&D sector. For simplification, in this section we will focus on the household and production sectors. We will introduce formally the R&D sector in the following sections that deal with the dynamics of the model.

In each period and each country, a set of endogenous variables are determined as the result of exogenous and state variables (see appendix A for the list of variables).

The exogenous variables are: the total population in country k (N_k) and the total amount of labour available in the country for the production sectors (L_k).

The first set of state variables ($l_{j,k}$) represents the quantity of labour in country k required to produce one unity of product j. Labour is assumed to be homogenous and workers move freely from one sector to the other, but labour is immobile between countries. The second set of state variables ($c_{j,k}$) represents the average per-capita quantity of product j that is demanded by the population in country k. The third set of state variables ($MK_{j,h,k}$) is the markup added to the price of commodity j produced in country k and consumed in country h. The other state variable is the wage rate (w_k) of country k expressed in the currency of country 1 ($w_1 = 1$).

Following Pasinetti (1993), the sets of state variables are assumed to be given at the beginning of each period in the model in the short-run. However, different from Pasinetti's framework, state variables are determined endogenously in the dynamic part of the model.

As mentioned, several macroeconomic variables are determined endogenously in each period.

Prices are assumed to reflect labour costs and markups. For each commodity j produced in country k and consumed in country h, the price of the commodity ($p_{j,h,k}$) is given by:

$$p_{j,h,k} = l_{j,k} w_k \, MK_{j,h,k} \qquad j=1,..,m;\ h,k=1,..,R \qquad (1)$$

Now let us turn to the determination of the quantities produced. The preferences of a representative consumer of a country k are already defined by the coefficients of consumption per capita $c_{j,k}$ ($j=1,\ldots,m$). What is not determined is the demand of that good j that is produced in a particular country h ($h=1,\ldots,R$). We represent that demand by $c_{j,k,h}$, which refers to the consumption per capita of good j in country k and produced in country h.

The consumption per capita of a commodity j in country k is the sum of the domestic consumption of commodity j that is locally produced and of commodity j that is imported:

$$c_{j,k} = \sum_{h=1}^{R} c_{j,k,h} \tag{2}$$

On the production side, the quantity of commodity j produced domestically in country k is equal to the domestic and foreign demands, and is given by:

$$Q_{j,k} = \sum_{h=1}^{R} c_{j,h,k} N_h \tag{3}$$

There are many possible ways to divide the consumption per-capita of commodity j ($c_{j,k}$) into the consumption of domestic and imported varieties of that product. Therefore, the model has to answer how the consumption per capita of a commodity in a country is divided between varieties produced domestically and varieties that are imported from the different countries.

Usual approaches follow Ricardo's (1821) principle of comparative advantage and result in full-specialization of exports (e.g. Andersen, 2001; Araujo and Teixeira, 2004), which does not replicate the empirical pattern of diversification. Other models use different mechanisms to allow for non-full-specialization, such as the use of export function following Thirlwall (1979) in which shares of domestic and imported goods are a function of the ratio of the prices in each country (e.g. Araujo, 2013, and Araujo and Trigg, 2015), but they are two-country models, as opposed to a multi-country model that can be used to study the pattern of diversification of several countries trading.

I adopt two mechanisms to avoid *ex-ante* full-specialization. The first is the markup prices, which is consistent with producers that have different labour costs competing in the same market by adopting markups that ensure that they meet the prevailing market price. The second mechanism is inspired by the World Trade Model proposed by Duchin (2005), using linear programming to determine the quantities produced for domestic and foreign consumption under the constraint of limited availability of labour.

The following linear programming is used to determine quantities. It minimizes expenditure given consumer preferences:

$$\text{Minimize} \quad \sum_{k=1}^{R} \left(\sum_{j=1}^{m} \left(\sum_{h=1}^{R} c_{j,k,h} \; N_k \; p_{j,k,h} \right) \right) \tag{4}$$

subject to:

a) The sum of quantity produced of a commodity j in all countries is the same as the sum of the quantity consumed of that commodity in all countries:

$$\sum_{h=1}^{R} \left(\sum_{k=1}^{R} c_{j,k,h} N_k \right) = \sum_{k=1}^{R} c_{j,k} N_k \tag{5}$$

b) The maximum total labour employed in each country is lower or equal than the total labour available in that country:

$$\sum_{j=1}^{m} Q_{j,k} \, l_{j,k} \leq L_k \qquad k=1,\ldots,R$$

$$\sum_{j=1}^{m} l_{j,k} \left(\sum_{h=1}^{R} c_{j,h,k} N_h \right) \leq L_k \qquad k=1,\ldots,R \tag{6}$$

c) For each country k the consumption per capita of a commodity j is the sum of the domestic consumption of commodity j that is locally produced and of commodity j that is imported, which is formulated in Eq. 2 and it is rearranged and restated below for convenience:

$$\sum_{h=1}^{R} c_{j,k,h} = c_{j,k}$$

d) And all quantities are non-negative numbers:

$$c_{j,k,h} \geq 0 \qquad j=1,..,m;\; h,k=1,\ldots,R \tag{7}$$

The linear programming above does not provide a unique result when two or more countries produce the same variety of a good with the exact same lowest price and without being constrained by the amount of labour available. In that situation, any combination of quantities of that good results in the lowest expenditure. To address that, we assume that these countries have the same market share in the production of the good with the lowest price.

The other endogenous macroeconomic variables of the model (employment, income, expenditure and balance of payments) follows Pasinetti's (1993) framework.

For each country *k*, employment in each sector *j* is equal to the labour required to produce the quantity of commodity *j* produced domestically:

$$E_{j,k} = l_{j,k} Q_{j,k} \qquad j=1,..,m;\ k=1,\dots,R \qquad (8)$$

At the macroeconomic level, total employment is the sum of employment in each sector of the economy:

$$E_k = \sum_{j=1}^{m} E_{j,k} \qquad k=1,\dots,R \qquad (9)$$

The output by sector ($Y_{j,k}$) is given by the price of the commodity multiplied by the quantity produced:

$$Y_{j,k} = \sum_{h=1}^{R} c_{j,h,k} N_h \ p_{j,h,k} \qquad j=1,..,m;\ k=1,\dots,R \qquad (10)$$

Total output of the economy *k* is the sum of the outputs of the individual sectors:

$$Y_k = \sum_{j=1}^{m} Y_{j,k} \qquad k=1,\dots,R \qquad (11)$$

Which divided by the population gives the output per capita in that country:

$$y_k = \frac{Y_k}{N_k} \qquad k=1,\dots,R \qquad (12)$$

Similarly, expenditure in country *k* on commodity *j* ($Exp_{j,k}$) is given by the consumption of domestic and imported varieties of the commodity multiplied by their prices:

$$Exp_{j,k} = \sum_{h=1}^{R} c_{j,k,h} N_k \, p_{j,k,h} \qquad j=1,..,m;\ k=1,\dots,R \qquad (13)$$

Total expenditure in country *k* is the sum of the expenditure on all products:

$$Exp_k = \sum_{j=1}^{m} Exp_{j,k} \qquad k=1,\dots,R \qquad (14)$$

The per-capita expenditure in country *k* is obtained by dividing the total expenditure by the population of the country:

$$exp_k = \frac{Exp_k}{N_k} \qquad k=1,\dots,R \qquad (15)$$

4.2 Wage rate

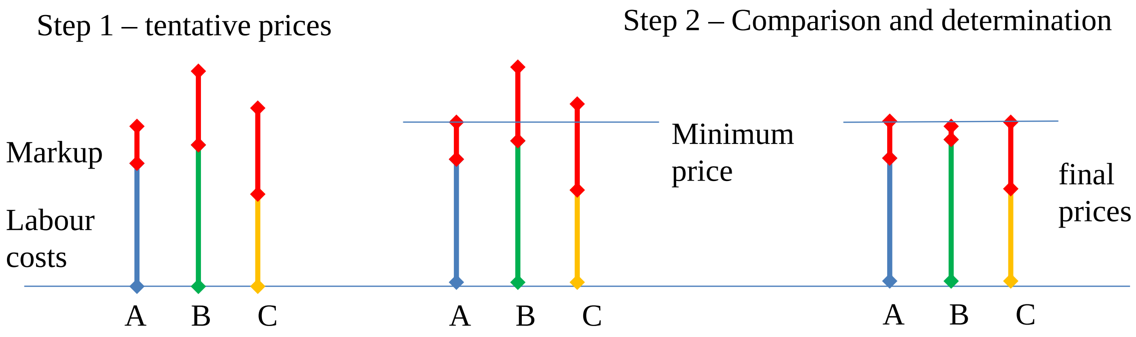
In the model we assume that nominal wage rates endogenously reflect the average productivity of the economy (Pasinetti, 1993). In the model we take the wage rate of country 1 as the unit and then we measure all the prices and wage rates in that currency. At each period the wage rates in other countries are given by the wage rate of country 1 multiplied by the ratio of average labour coefficients in both countries weighted by the employment shares:

$$w_k = w_1 \frac{\left(\sum_{j=1}^{m} l_{j,1} E_{j,1} / E_1\right)}{\left(\sum_{j=1}^{m} l_{j,k} E_{j,k} / E_k\right)} \qquad k=\{2,3,..,R\} \qquad (16)$$

4.3 Change in markup prices

We assume monopolistic competition in domestic and international markets with markup prices for the products (e.g. Krugman, 1979). The mechanism to determine markup prices is comprised of two steps that are carried out during the transition from one period of time to the next. This mechanism is illustrated in Figure 3 considering three countries (A, B, C) and a single sector. In the figure, a red line represents a markup. Unit labour costs of production of the commodity in each country/sector are represented by the colours blue (country A), green (country B) and yellow (country C).

Figure 3
Illustration of mechanism to determine markup prices



The first step is the setup of tentative prices by each sector in each country and targeting each market. Tentative prices are composed by the unit labour cost of production and the markup that was used in the previous period of time. The second step is the comparison and determination of price, in which sectors by country compare their prices with their competition from other countries and, if the latter has lower prices, reduce the markup to match the competitor's price. The result is that competition drives prices down.

4.4 Technological change

Technical progress is assumed in the model to be the result of four different processes: 1) introduction of new techniques in the production of an existing product (process innovation); 2) production of a new good that does not yet exist in any country (product innovation); 3) the introduction of techniques that are new to the country's production but not new to the global economy (process emulation); and 4) introduction of a product that is new to the country but not new to the world (product emulation).

The emergence or disappearance of economic sectors is restricted to the moment of passing from one period of time to the other.

Adjacent possible for creation of path dependency of innovation

We adopt the concept of adjacent possible as proposed by Kauffman (2008) to create the path dependency of the innovation process.

Suppose at each passage of time, and in relation to each country k, there is an adjacent possible (AD_k) of potential new products or processes that could be created by the combination of existing set of technologies in the country.

We assume that there is a parameter u (for useful) that represents the odds that the potential new product is useful to fulfil a human need or not. Lists of useful (List 1) and not useful (List 2) potential products are generated through this process to keep consistency. If a potential new product j is considered not useful when it is a member of an adjacent possible of a country k, it will also be considered not useful in the adjacent possible of other countries.

To keep the computer simulation of the model manageable, we assume that potential new products are the result of combination of only two technologies already used in the production.[4]

R&D sector

We assume that process and product innovation and emulation are carried out through R&D and are assumed to be funded by the sum of the markup prices. Therefore, the number of people engaged in R&D is constrained by the amount of surplus obtained by the production sectors in selling products with markup (*SP*). If productivity in the country is higher than in other countries and a high sum of markup is obtained, then the productive sectors can fund a large R&D effort. When productivity is low and country's exports are few and with lower markup, then the R&D effort is limited.

$$SP_k(t) = \sum_{h=1}^{R} \sum_{j=1}^{m} \big((p_{j,h,k}(t) - l_{j,k}(t) w_k(t)) Q_{j,k}(t)/w_k(t)\big)$$

$$SP_k(t) = \sum_{h=1}^{R} \sum_{j=1}^{m} \big(l_{j,k}(t)(MK_{j,h,k}(t) - 1) Q_{j,k}(t)\big) \tag{17}$$

The number of people engaged in R&D is also constrained by the availability of labour to participate in that activity (*LA*), which is drawn from the labour that is not engaged in production:

$$LA_k(t) = L_k(t) - E_k(t) \tag{18}$$

The share of labour engaged in R&D (ϵ_k) is then given by:

$$\epsilon_k(t) = \frac{min(SP_k(t), LA_k(t))}{L_k(t)} \tag{19}$$

We assume that out of that group of people, a share of them is devoted to research towards a new product ($\partial_k^{product}$) (either to the country or to the world) and another share is devoted to find a new and more productive way to produce an existing product ($\partial_k^{process}$):

$$\partial_k^{product}(t) + \partial_k^{process}(t) = 1 \tag{20}$$

$$0 \leq \partial_k^{product}(t) \leq 1;\ 0 \leq \partial_k^{process}(t) \leq 1 \tag{21}$$

We assume that the shares of research dedicated to find a new product or a new process are endogenous to the model and a function of the share of the labour force that is employed. The assumption is that R&D effort towards process innovation increase with employment to reduce the labour requirement in the existing production base, while, on the other hand, the effort towards product innovation increases to create new sources of demand and employment when labour participation reduces. Therefore, the higher the level of employment, the higher the number of people engaged in finding more productive ways to produce the existing products and the lower the number of people trying to find new products, and vice-versa:

$$\partial_k^{process}(t) = E_k(t)/L_k(t) \tag{22}$$

$$\partial_k^{product}(t) = 1 - E_k(t)/L_k(t) \tag{23}$$

Out of the group of people engaged in finding a new product, a share of them is devoted to research towards product innovation ($\sigma_k^{product}$) and another share is devoted to emulation ($\sigma_k^{product\_emulation}$):

$$\sigma_k^{product}(t) + \sigma_k^{product\_emulation}(t) = 1 \tag{24}$$

$$0 \leq \sigma_k^{product}(t) \leq 1;\ 0 \leq \sigma_k^{product\_emulation}(t) \leq 1 \tag{25}$$

4 This decision is mainly due to hardware limitations and is not related to the model.

We assume that the shares of research dedicated to product innovation and emulation are endogenous to the model in the following way:

$$\sigma_k^{product}(t) = m_k(t)/m(t) \tag{26}$$

$$\sigma_k^{product\_emulation}(t) = 1 - m_k(t)/m(t) \tag{27}$$

Where m_k is the number of types of commodities produced in country *k* and *m* is the combined number of different types of commodities traded by all countries. Therefore, if a country already produces all types of commodities that exist, its research related to finding new products will be totally devoted to product innovation.

Similarly, out of the researchers working to discover a more productive process of production, a share is devoted to research towards process innovation ($\sigma_k^{process}$) and another share is devoted to process emulation ($\sigma_k^{process\_emulation}$):

$$\sigma_k^{process}(t) + \sigma_k^{process\_emulation}(t) = 1 \tag{28}$$

$$0 \leq \sigma_k^{process}(t) \leq 1;\ 0 \leq \sigma_k^{process\_emulation}(t) \leq 1 \tag{29}$$

The shares of research dedicated to process innovation and process emulation are assumed to be endogenous to the model as follows:

$$\sigma_k^{process}(t) = TF_k(t)/m_k(t) \tag{30}$$

$$\sigma_k^{process\_emulation}(t) = 1 - TF_k(t)/m_k(t) \tag{31}$$

Where TF_k is the number of sectors in production in country *k* that are operating at the technological frontier, meaning that they have the highest productivity when compared with similar sectors in other countries. Therefore, if all sectors are operation at the technological frontier, its research related to finding more productive processes will be totally devoted to process innovation.

Process innovation

Process innovation, product innovation and emulation are included in the model using a mechanism similar to that proposed by Andersen (2001). In the case of process innovation, in each country *k* the outcome of the work of one person engaged towards process innovation takes the form of a Poisson process with the arrival rate of new process given by $\lambda_k^{process}$.

The effective arrival rate of a new process in country *k* and sector *j* is a function of the number of people in the R&D sector of that country who are engaged in process innovation ($\epsilon_k \partial_k^{process}\ \sigma_k^{process} N_k$). The effective arrival rate is also a function of the share of employment in that sector ($E_{j,k}/E_k$), which is assumed to create an incentive to augment labour.

Therefore, in each country *k* and sector *j*, the number of new processes in each period of time is given by:

$$X_{j,k} \sim P(\epsilon_k \partial_k^{process}\ \sigma_k^{process} N_k (E_{j,k}/E_k)\ \lambda_k^{process}) \tag{32}$$

When process innovation happens in a sector *j*, we consider that the labour coefficient of that sector is reduced by multiplying for a factor ($0 < rdc_{j,k} < 1$):

$$l_{j,k}(t) = rdc_{j,k}\ l_{j,k}(t-1) \tag{33}$$

For simplification, we assume that $rdc_{j,k}$ is drawn from a standard uniform distribution:

$$rdc_{j,k} \sim U(\beta_1, 1), \quad 0 < \beta_1 < 1 \tag{34}$$

Where β_1 is a parameter of the model.

Process emulation

Process emulation is modelled in a similar way as process innovation. We consider that one person engaged towards process emulation in country *k* would find a new process at an arrival rate of $\lambda_k^{process\_emulation}$ that takes the form of a Poisson process.

The effective arrival rate of a new process through emulation in country *k* and sector *j* is a function of the number of people in the R&D sector of that country who are engaged in process emulation ($\epsilon_k \partial_k^{process} \sigma_k^{process\_emulation} N_k$) and the share of employment in the sector ($E_{j,k}/E_k$). A sector would only undergo process emulation if the sector is lagging behind the technological frontier.

Therefore, in each country *k* and sector *j*, the number of new processes through process emulation in each period of time is given by:

$$X_{j,k} \sim \begin{cases} P(\epsilon_k \partial_k^{process} \sigma_k^{process\_emulation} N_k (E_{j,k}/E_k)\, \lambda_k^{process\_emulation}) \text{ if lagging behind} \\ 0, \text{otherwise} \end{cases} \tag{35}$$

When process emulation happens in a sector *j*, we consider that the sector adopts the technologies of the frontier country. Hence, the labour coefficient takes the value of the coefficient in the country *h* and sector *j* that was emulated:

$$l_{j,k}(t) = l_{j,h}(t) \tag{36}$$

Product innovation

Regarding product innovation, we assume that in each country *k* the outcome of the work of one person engaged towards product innovation takes the form of a Poisson process with the arrival rate of new sector given by $\lambda_k^{product}$.

The effective arrival rate of new products in the economy is rescaled by the number of researchers that are engaged in product innovation, and hence is given by $\epsilon_k \partial_k^{product} \sigma_k^{product} N_k \lambda_k^{product}$.

Therefore, in each country *k* the number of new products in each time phase is given by:

$$X_k \sim P(\epsilon_k \partial_k^{product} \sigma_k^{product} N_k \lambda_k^{product}) \tag{37}$$

In the case that a new product emerges through that Poisson process, we assume that a new production sector could be established to produce that commodity. For simplification, we assume that the labour coefficient of the new sector (l_{new}) is given by the average of the labour coefficients of the production sectors in activity in the economy in the previous period:

$$l_{new,k}(t) = \frac{\sum_{j=1}^{m_k} l_{j,k}(t-1)}{m_k(t-1)} \tag{38}$$

Note that the new product is created but it does not mean that a new sector will automatically be created to produce that commodity. A new sector will start to operate if there is already a potential demand for the new product. In other words, a new sector starts if there is at least one country in which the level of income per-capita is sufficient to allow a potential demand for that new product. If no potential demand exists then production of the new product will be on hold waiting for the demand.

Product emulation

Product emulation is the innovative research and development process required to imitate and adapt to domestic conditions the production that already exist in another country. Similar to product innovation, we assume that in each country k the outcome of the work of one person engaged towards emulation takes the form of a Poisson process with the arrival rate of new sector given by $\lambda_k^{emulation}$. The effective arrival rate of a new emulation in the economy is rescaled by a share φ of the number of researchers that are engaged in the process of emulation, which total is given by $(\epsilon_k \partial_k^{product}\ \sigma_k^{emulation} N_k)$.

The share φ is assumed to be proportional to the emulation opportunities of the sector, which is defined here as the increase in the share of the demand in that sector in the total demand for products that are new to the country. The idea is that the higher the increase in demand, the higher the opportunities for new entrants to position themselves in the market. Therefore, if there are two products for which the country can engage in emulation, the higher share of the researchers will be working towards the emulation of the production of the sector that is experiencing the higher increase in demand. The share φ is calculated as follows:

$$\varphi_{j,c}(t) = \frac{\sum_{k=1}^{R} Y_{j,k}(t-1) - \sum_{k=1}^{R} Y_{j,k}(t-2)}{\sum_{i=1}^{m} \sum_{k \neq c}^{R} Y_{i,k}(t-1) - \sum_{i=1}^{m} \sum_{k \neq c}^{R} Y_{i,k}(t-2)} \tag{39}$$

Where $\sum_{k=1}^{R} Y_{j,k}$ is the total output in monetary terms of production of commodity j, which is the same as the total demand in monetary terms, and $\sum_{i=1}^{m} \sum_{k \neq c}^{R} Y_{i,k}$ is the sum of output of the m sectors that are the result of product innovation in countries other than country c and are not produced in that country. The rule above describes an economic incentive that directs the emulation efforts.

For the process of emulation to start, it is necessary that a new product has previously emerged in other country. That would trigger the research towards emulation taking the form of the Poisson process. Thus in each time period and in each country k the number of emulations of a particular new sector j is given by:

$$X_{j,k}(t) \sim \begin{cases} P\left(\epsilon_k \partial_k^{product}\ \sigma_k^{emulation} N_k \varphi_{j,c} \lambda_k^{emulation}\right), & \text{if } j \text{ is a new sector in any other country} \\ 0, \ otherwise & \end{cases} \tag{40}$$

For every country k and for every emulated sector j, we assume that its labour coefficient at the time of the emulation (t'') is the same as the labour coefficient of that sector at the time that it was created (t') in the economy of country h that initially introduced the commodity:

$$l_{j,k}(t'') = l_{j,h}(t') \tag{41}$$

4.5 Change in consumption patterns

Let us start the discussion of change in consumption patterns by looking at Pasinetti's (1993) model, which formalizes these changes in a general form as follows:

$$c_{j,k}(t) = c_{j,k}(t-1)e^{r_{j,k}} \tag{42}$$

Where $r_{j,k}(t)$ is the rate of change in consumption, which is assumed to be constant during that period of time.

Similar to Pasinetti's (1993) framework, consumption patterns changes according to a generalized version of Engel's law that can be summarized by the following three empirical regularities: i) there is an hierarchy of preferences of consumption in which increases in real income result in faster increases in demand for some goods than for others; ii) new products trigger changes in the composition of consumption; and iii) there is a saturation level for the consumption of any product (Pasinetti, 1993, pp.39-40).

To implement fact (i) related to the hierarchy of preferences of consumption, we assume that the commodities are ordered from those that satisfy the most to those that satisfy the least essential needs.

Therefore, we enforce a decreasing order on the rate of changes ($r_{j,k}$) in each country in each period of time:

$$r_{1,k} \geq r_{2,k} \geq \cdots \geq r_{m_k,k} \tag{43}$$

Such and ordering implies that when consumption is increasing, it will increase faster for the more essential goods. On the other hand, when consumption is decreasing, it will decrease faster for the less essential goods.

To implement the empirical fact (iii), related to saturation of demand, we assume that for each commodity *j* there is a maximum amount for the consumption per capita of that commodity given by $maxc_j$. For simplification, we consider that such maximum value is the same in all countries. Therefore, in each country *k*, the consumption per capita $c_{j,k}$ of commodity *j* cannot grow beyond $maxc_j$ and the equation (42) is rewritten as:

$$\forall j \; \exists \; maxc_j, \;\; c_{j,k}(t) = \min(c_{j,k}(t-1)e^{r_{j,k}}, maxc_j) \tag{44}$$

For the computer-based implementation of the model, we assume that $maxc_j$ is higher than the maximum value at time 1 (here represented by a new variable MAX_j) of the consumption per capita of commodity *j* when considering all countries:

$$maxc_j > MAX_j = \max(c_{j,1}(1), c_{j,2}(1), \ldots, c_{j,m_k}(1)) \tag{45}$$

For simplification, we assume that $maxc_j$ is drawn from a standard uniform distribution:

$$maxc_j \sim U(MAX_j, \beta \, MAX_j \,) \tag{46}$$

Where β is a parameter that represents the maximum increase that the consumption per capita may take. For example, if at time 1 the maximum value of consumption per capita of commodity *j* is 10 units ($MAX_j = 10$), and if the parameter β is set to the value 2, then the saturation point of that commodity ($maxc_j$) will be a stochastic value between 10 and 20 units.

Equations (43) to (46) formalize the Pasinettian framework, the next step is to endogenize the change of total consumption expenditures. Here we adopt the Keynesian view of consumer demand. More specifically, we adopt a mechanism similar to Clower's (1965) "dual-decision hypothesis", in which households use a two-steps decision process and decide on their demand for goods only after their actual incomes are known. The mechanism works as follows. First, households receive their income and, based on that and on the current prices of products, decide on consumption preferences for the next period. If the income received is lower than the latest expenditure, then people will have a lower expectation related to how much their income will be able to fulfil their consumption in the next period, and they would decide to consume less as consequence. If, on the other hand, the income received is higher than latest expenditure, then people will have a higher expectation for the purchasing power of their income in the next period and would decide to consume more. When households actually consume in the following period, firms decide on the level of employment to fulfil that demand, which determines the next income. In the model, the actual consumption change for each product is given stochastically based on a uniform distribution, but the direction of the change is calculated through that "dual-decision hypothesis" process and, therefore, it is endogenous to the model.

In addition, we follow the principle in the agent-based literature (as discussed in Backhouse and Boianovsky, 2013) and assume heterogeneous behaviour of consumers who follow rules of thumb. Therefore, we consider that the aggregated result of the behaviour described in the paragraphs above is a stochastic change in the consumption per capita.

The result is that when income is higher than expenditure in the previous period, the aggregated behaviour of consumers is to increase their consumption in the current period, and vice-versa.

Therefore, here we assume that the signal (positive or negative) of the growth rate of consumption r is determined endogenously and it is given as follows:

$$\begin{cases} r_{j,k}(t) > 0, if\, exp_k(t-1) < y_k(t-1) \\ r_{j,k}(t) < 0, if\, exp_k(t-1) > y_k(t-1) \end{cases} \tag{47}$$

For simplicity, we also assume that r is drawn from standard uniform distributions:

$$\begin{cases} r_{j,k} \sim U(0, \max(r)), if\, exp_k(t-1) < y_k(t-1) \\ r_{j,k} \sim U(-\max(r), 0), if\, exp_k(t-1) > y_k(t-1) \\ r_{j,k} = 0,\ if\, exp_k(t-1) = y_k(t-1) \end{cases} \tag{48}$$

Where $\max(r)$ is a parameter that indicates the maximum absolute rate of change r in all sectors and in all countries.

Note that the difference between total income and total expenditure is the same as the value of the balance of payments of an economy. Therefore, this mechanism also serves the function to keep the balance of payments moving towards zero in the long run. With that mechanism the model does not need to enforce balance-of-payment equilibrium by adding constraints to consumption or supply levels in each economy.

Now let us get back to the empirical fact (ii) in Pasinetti's (1993) framework that states that a variation in the composition of consumption may occur as a consequence of the introduction of new products. First we assume that the consumption of a new commodity *j* will only occur if the level of income per capita is higher than a certain value (α), below which there is no consumption of that commodity. We assume that this floor in relation to the demand of a particular commodity is the same in all countries. Thus the consumption of a new commodity *j* will start at a time (t''), at the same time or posterior to the creation of the new product at time (t'), according to the following :

$$c_{j,k}(t'') \quad \begin{cases} > 0, if\ y_k(t''-1) \geq \alpha \\ = 0\ ,\ \ otherwise \end{cases} \tag{49}$$

Here we assume that the emergence of a new commodity results in an exogenous small one-time change in the pattern of consumption of all other commodities, due to the complementarities or substitutions that the new product allows. That is the "creative destruction" process in action. Let's define $s_{i,j}$ as the substitution and complementarity effect of the emergence of the new product *i* on the consumption of an existing commodity *j*. For simplicity, we also assume that $s_{i,j}$ is drawn from a standard uniform distribution with the maximum change given by the parameter κ:

$$s_{i,j} \sim U(1-\kappa, 1+\kappa) \tag{50}$$

Therefore in each country *k*, by the time of the introduction of the new product *i*, the coefficient of consumption of existing product *j* is affected in the following one-time change (revising equation 44):

$$c_{j,k}(t) = \min(s_{i,j}\ c_{j,k}(t-1)e^{r_{j,k}}, maxc_j) \tag{51}$$

Similarly, the saturation point of commodity *j* is affected with the change:

$$maxc_j(t) = s_{i,j}\ maxc_j(t-1) \tag{52}$$

5 Replication of the stylized facts

This section presents the results of the experiment to verify if the model is able to reproduce the patterns of diversification observed in the actual trade data.

The model indeed replicates the three stylized facts: (1) diversification is associated with higher total GDP, (2) diversification is associated with lower average ubiquity of exports, and (3) diversification is path dependent, which is true in the model by design (making technologies emerge through the combination of existing technologies).

To perform that test we generate simulations with 30 countries, 10 initial products and the period of 100 time units. All countries start the simulations with exactly the same parameters and have the same values for all macroeconomic indicators. We run simulations of the model 100 times to test different realizations of the stochastic process that uses the following set of initial value configuration and parameters:

- labour coefficient $l_1 = l_2 = \cdots = l_{30} = \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5\}$;
- coefficient of consumption per capita $c_1 = c_2 = \cdots = c_{30} = \{0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01\}$;
- wage rate in country 1 $w_1 = \$\ 1$;
- $\beta = 2$; saturation of consumption not higher than 100% above the initial consumption per capita;
- $\lambda_k^{product} = \lambda_k^{process} = 1/200$; in any country *k* (*k*=1,2,3…30) one researcher would find a new product or a new process on average once every 200 periods of time;
- $\lambda_k^{product\_emulation} = \lambda_k^{process\_emulation} = 1/100$; in any country *k* (*k*=1,2,3…30) one researcher engaged in emulation would find a new way to emulate a product once every 100 periods of time;
- *u* = 0.2; 2 out of 10 potential new products as combination of existing technologies are useful.

The set of figures shown in the next pages present the result of one of the 100 runs of the simulation. Figure 4 shows that the model replicates the stylized fact 1. The horizontal axis of the graph shows the diversification of each country at the end of the simulation run (time 100) measured by the number of products exported, and the vertical axis shows the total GDP in each country measured by the currency of country 1 (the vertical axis is shown in logarithmic scale). Each circle in the graph represents a country. Some circles are shown overlapping with each other in the figure, particularly visible for countries with lower levels of diversification, which indicates that countries have about the same level of GDP for similar levels of diversification. Also noticeable is the large income inequality between more diversified countries and the others. The total GDP of each of the two more diversified countries (both exporting 26 products) are 2 to 3 times higher than the total GDP of the third more diversified country and 200 to 300 times higher than the less diversified countries (which export the same initial 10 products).

Figure 4

Association between diversification and GDP (time=100)

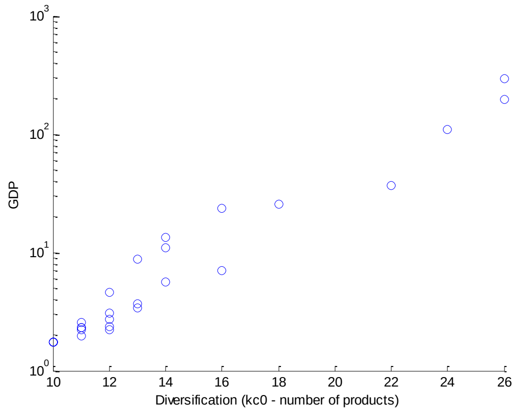


Figure 5

Association between diversification and average ubiquity of exports (time=100)

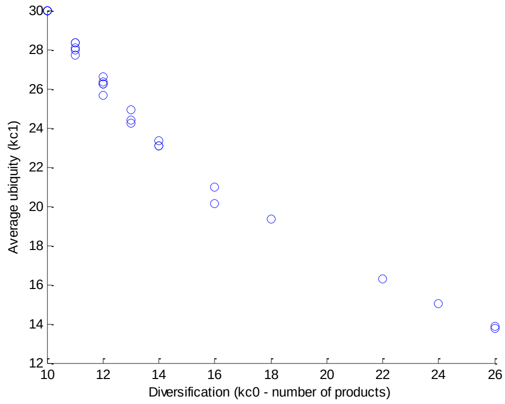


Figure 6

Association between diversification and employment (time=100)

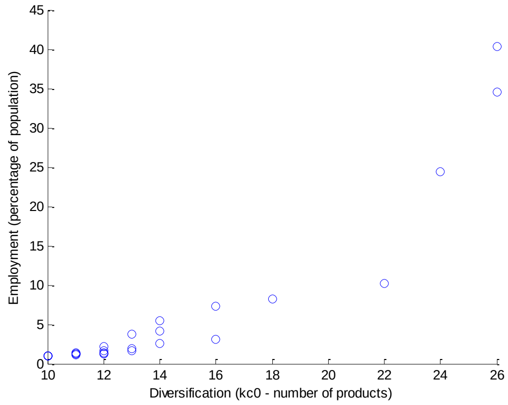


Figure 7

Association between diversification and consumption per capita (time=100)

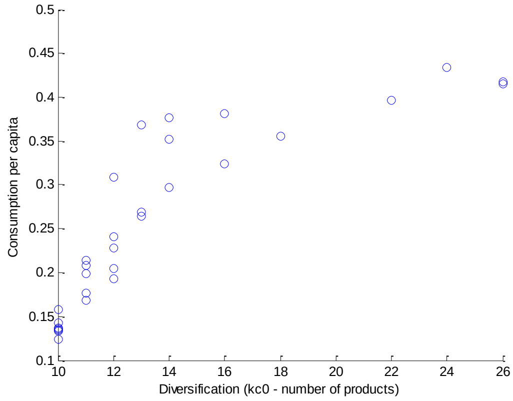


Figure 8

Association between diversification and average labour coefficient (time=100)

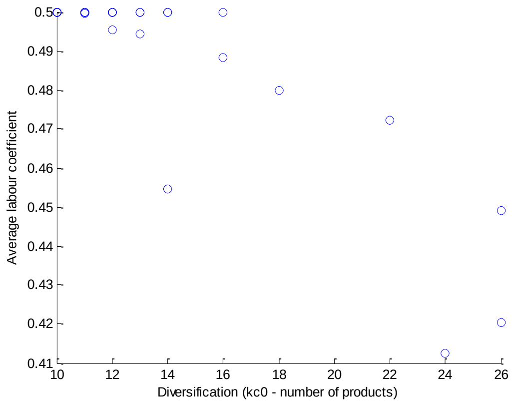


Figure 5 shows that the model replicates stylized fact 2, the negative association between diversification (horizontal axis) and the average ubiquity of the exports of a country (vertical axis). The graph shows that the value of average ubiquity of the exports of the more diversified countries (14) is less than half of the less diversified countries (30), which indicates that on average the more diversified countries face half of the competition faced by the less diversified countries.

Figure 6 shows that diversification is also positively associated with employment levels in percentage terms (vertical axis) at the end of the run. The graph presents a pattern that resembles the association between diversification and total GDP (Figure 4). That is expected given that total GDP is calculated as the production of a country times the prices of its products sold in each market, and one component of price is the labour cost (the other is the markup). Therefore, the higher the level of employment, the higher the labour costs and the higher the total GDP of a country. Note that the employment levels of the two more diversified countries (40% and 35%, respectively) are closer than the GDP levels of these two countries, which indicates that significant contributors for the differences in GDP levels is the differences in wage rates (as result of differences in average productivity) and differences in average markup levels of the products of each country.

Figure 7 presents the positive association between diversification and consumption per capita. It is noticeable that as diversification increases, the differences in the levels of diversification between countries, and consequently the differences in income (as seen in Figure 4), result in smaller differences in expenditure. The reason for that is the saturation of demand of commodities in the economy. Above certain levels of income, the consumption per capita of the products that have reached saturation remains constant.

Figure 8 shows the negative association between diversification and the average labour coefficient. The circles at the top of the graph represent countries that have not experienced process innovation; the average labour coefficient in these economies has remained at the value of 0.5. Some of these countries have diversified, hence have experienced product innovation, but that has not changed the average labour coefficient due to the simplification adopted in the model in which a new sector that emerges through product innovation has the labour coefficient equal to the average labour coefficient in the economy. Other circles in the graph represent countries in which sectors have undergone process innovation. The more diversified the country, the higher the level of employment and the higher the share of R&D dedicated to process innovation in comparison with product innovation. Therefore, the higher the probability that more productive sets of technologies are adopted by sectors in the economy, reducing the average labour coefficient. The result is a tendency of lower average labour coefficient associated with higher levels of diversification. We should note that innovation is the result of a stochastic process; therefore, it is possible that, during the run, process innovation occurs more often in a country and less often in others, all things being equal.

The summary statistics of the 100 runs of the simulation are presented in Table 1. In terms of diversification, on average the countries that are less diversified at the end of a run of the simulation have not undergone any product innovation or product emulation. They have completed the runs with the same 10 sectors that they had at the beginning. The median country on average ends the run with 11 sectors, which shows that half of the countries are not able to increase much their diversification. On the other hand, more diversified countries have on average 33 sectors exporting at the end of a run of the simulation.

Table 1
Summary statistics of 100 runs of simulation

		Mean	Standard deviation
Diversification	Minimum	10	0.14
	Maximum	33	4.56
	Median	11	0.58
Relationship between logarithm of diversification and logarithm of GDP ($\ln(k_{c0})$ x $\ln(GDP)$)			
	Slope	4.61	0.46
	R-squared	0.94	0.03
Relationship between logarithm of diversification and logarithm of average ubiquity ($\ln(k_{c0})$ x $\ln(k_{c1})$)			
	Slope	-0.85	0.03
	R-squared	1	0

The table also presents the slope of the least-squares regression line that summarizes the relationship between the logarithm of the level of diversification of a country ($k_{c,0}$) and the logarithm of GDP. The empirical evidence shows a positive slope for that relationship, which is replicated by the model as indicated by the positive value 4.61. Similarly, the table shows the slope of the least-squares regression line related to the logarithm of the level of diversification ($k_{c,0}$) and the logarithm of the average ubiquity of exports ($k_{c,1}$). A negative value (-0.85) is consistent with the empirical relationship in which a higher level of diversification is associated with a lower level of average ubiquity of exports.

A sensitivity analysis of the model is presented in Appendix B. It shows that the results of the model are not affected by the changes in the initial values of the key parameters related to economic diversification: the arrival rates of innovation and emulation.

6 Catch-up strategies of poorer countries

This section presents the analysis of possible strategies to facilitate the catch-up and diversification of poorer countries. We run 100 simulations of 50 time periods for each strategy, with 10 countries, 6 initial products and the same initial configuration as used in the analysis in the Appendix B. For each run of a simulation, data was collected at the middle of the run (time=25) and at the end (time=50). The data recorded was related to the country that at time 25 was the least diversified among the 10 countries. A catch-up strategy is applied only to that country that was lagging behind, from the time 25 to the end of the run, to verify if at the end of the run that country was able to diversify and catch up with the other countries.

This section presents the results of the analysis of one benchmark strategy and seven catch-up strategies (Table 2). The benchmark strategy is the normal functioning of the model, which is used to compare with the other strategies. The next three strategies are based on focusing in only one process of technological change: the 'focus on emulation' strategy concentrates all the R&D effort in product emulation, the 'focus on product innovation' strategy concentrates R&D on the effort to find a new product; and the 'focus on process innovation 'strategy concentrates all R&D on finding a more productive way to produce an existing product.

Table 2
Strategies used in the analysis

Strategies	Description
Benchmark	Normal run of the model. It is used as benchmark.
Focus on product emulation	R&D concentrates in product emulation
Focus on product innovation	R&D concentrates in product innovation
Focus on process innovation	R&D concentrates in process innovation
Target more complex products	Product emulation target the products that have higher than average product complexity
Undervalued currency by 10%	The wage rate is reduced by 10% relative to the international currency
Focus on product emulation & Target more complex products	R&D concentrates in product emulation, which target products with above average product complexity
Focus on product emulation & Target more complex products & undervalued	R&D concentrates in product emulation, which target products with above average product complexity, and higher competitiveness created by a 10% undervalued wage relative to the international currency

The fourth strategy is the same used in several studies in the literature of economic complexity to identify products with high opportunities for diversification (Hausmann and Klinger, 2008; Freitas and Salvado, 2008; Neves, 2012; Freitas, Salvado et al., 2013; Freire, 2013a, 2013b; ESCAP, 2014, 2015). For each country, the average product complexity of the exports is calculated (see Appendix C for the details). The strategy is to focus product emulation on the products that have complexity above the average levels of the product complexity of the country that was lagging behind. The idea is that the more complex products would offer higher opportunities for further diversification since they are associated with countries that are more diversified and have a less ubiquitous production.

A fifth strategy is to keep the currency of the country that was lagging behind undervalued (by 10%) to increase the competitiveness of its exports, which would increase the revenue and therefore the resources available to finance R&D. This strategy has the objective to illustrate the idea that an undervalued currency benefits poorer countries. That argument has been an important element in the Latin American structuralist literature and also in the context of the analysis of the Chinese growth process (e.g. Bresser-Pereira, 2012, Rodrik, 2008).

The sixth strategy is the combination of 'focus on emulation' and the strategy that target the emulation of products with above average complexity; while the seventh strategy adds the undervaluation of currency to that mixed strategy.

Each particular strategy was applied in four different configurations of the model in terms of the rates of arrival of product innovation and emulation. The objective is to show that the result of the different strategies are also affected when rates of product innovation increases and emulation is facilitated. The configurations used are as follows:

A. $\lambda_k^{product} = \frac{1}{100}$; $\lambda_k^{product_{emulation}} = \frac{1}{10}$: A researcher working in R&D would find one product innovation every 100 units of time and one product emulation every 10 units of time.

B. $\lambda_k^{product} = \frac{1}{200}$; $\lambda_k^{product_{emulation}} = \frac{1}{20}$: A researcher working in R&D would find one product innovation every 200 units of time and one product emulation every 20 units of time.

C. $\lambda_k^{product} = \frac{1}{100}$; $\lambda_k^{product_{emulation}} = \frac{1}{50}$: A researcher working in R&D would find one product innovation every 100 units of time and one product emulation every 50 units of time.

D. $\lambda_k^{product} = \frac{1}{200}$; $\lambda_k^{product_{emulation}} = \frac{1}{100}$: A researcher working in R&D would find one product innovation every 200 units of time and one product emulation every 100 units of time.

Figure 9 (A), (B) and (C) present a summary of the effects of each strategy, based on the average of the results of all runs of simulations for each of the configurations above. In these figures the strategies are lined up in the horizontal axis and the configurations in the vertical axis. Both are ordered in a way to make the contours in the figures look smother to facilitate the visualization. This order is purely a matter of choice and any other could be used. The order selected for the strategies was to line them up from the worst to the best strategy in terms of diversification at the end of the run, when considering faster product innovation and emulation (i.e. configuration A).

The measure used in Figure 9 (A) is the gain in diversification by using a particular strategy, when comparing with the performance of the other countries. The idea is that the strategies that perform better will facilitate the increase of diversification of the poorest countries faster than the increase of the diversification of the average country in the same period. Such strategies would allow the poorest country to catch up with the average country. This measure of relative gains in diversification is calculated by the difference of a normalized measure of diversification at the middle and at the end of the simulation runs (diversification of a country is normalized using z-scores, i.e. by subtracting the global average diversification and dividing by the standard deviation). A value of zero means that in both points in time the country had the same diversification in relation to the global average; hence there was no gain in relative terms. A positive value indicates that the country became more diversified than the average, while a negative value shows the opposite.

Figure 9 (B) and (C) present, respectively, the percentage increase in GDP and employment of the initially less diversified country as result of the adoption of the different strategies.

In the figures it is noticeable that the larger variation in the results is seen in relation to configurations A and B (more frequent emulation), in contrast to the lower variation shown in the configurations C and D (less frequent emulation). The reason for this pattern is because when emulation is facilitated in the less diversified country (for example through technology transfer), all strategies that use emulation (i.e. six out of the seven strategies tested not counting the benchmark) have their effects boosted by the increase in the frequency that a new emulation could emerge. That generates the larger variation of results.

When comparing the results related to configuration A and B, larger variation is seen in the results of configuration A (more frequent product innovation) than of configuration B (less frequent product innovation). The same pattern is seen when comparing the results of configuration C and D. The reason is because the strategies for catch-up rely on accelerating technological change in the less diversified countries, and when innovation (and for that matter emulation) happens at a slower pace, the effects of these strategies are less effective. These comparisons suggest that the differences among the strategies in terms of promoting diversification are more pronounced when there is more product innovation and emulation is facilitated.

These figures show that the strategies that the focus on process and product innovation underperform the benchmark scenario. The worst strategy is to focus all R&D effort towards process innovation, hindering all possibilities for diversification. The reason for such low performance is because all innovation is directed to improve labour productivity of existing products, which reduces prices, output and employment. The reduction in price could lead to higher demand, but that is limited by the saturation of consumption and by the shift of demand to new products that are created by other countries that carry out product innovation and emulation. Even if there is an increase in demand, it may not be all captured by the country that generated the process innovation because other countries may be able to reduce their markup to match the lower international price, keeping the market shares unchanged. The result is a decrease in diversification when compared with global average, lower GDP and lower employment when compared with the benchmark strategy.

In the real global economy, product innovation is usually carried out in more developed countries anyway, but process innovation is a strategy that we may see associated with industries in less diversified countries, particularly in sectors that are established through foreign direct investment and in which the business model relies heavily in lowering production costs of existing production. In that case, less diversified countries could be paradoxically worse off if its firms engage preferentially in process innovation. Therefore, these results suggest that policies that promote innovation and increase in productivity in less diversified countries may be in fact counterproductive if they target process innovation instead of product emulation as the innovation strategy.

Focus exclusively on product innovation does not help to catch up either. The economy of the country that is lagging behind is less diversified and has a smaller set of technologies available for combination and generation of new products. Most probably these new products that could emerge from that combination are not new to the world anyway. Given that all countries in the simulation start with the same set of products and underlying technologies, the adjacent possible of the less diversified country is likely to be composed by potential new products that are only new to the production base of that country; they were already created in other more innovative countries in the past. That strategy only makes this countries persist in a fruitless quest for products that are new to the world.

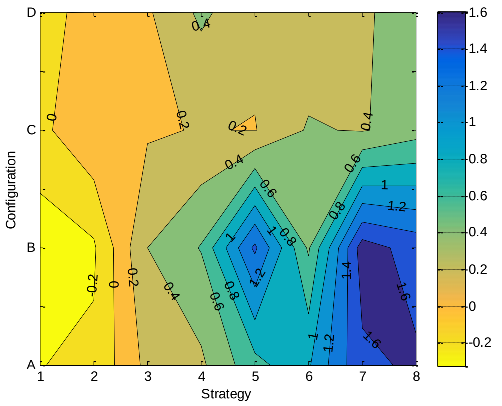
The other strategies tested are either considerable better or at least as good as the benchmark (strategies 4 to 8 in the figures).

Strategy number 4 is the one that undervalues of the currency by 10%. That strategy outperforms the benchmark in terms of increasing diversification, but results in similar levels of employment and lower GDP. An undervalued currency reduces the labour costs of exports and makes the less diversified country more competitive in the products that it is able to produce. Moreover, the country is able to produce and export some additional products that otherwise would have a price higher than the international price if not by the undervalued currency. That results in higher diversification than the benchmark scenario. However, the undervalued currency also makes import prices relatively higher than normal. People of the country would have to reduce their consumption, which would also affect the domestic production. Therefore, the gains in employment related to the additional sectors added to the economy are reduced by the lower employment due to lower consumption. The effect in output is negative given that prices of exports are reduced. It is important to note that in these simulations all countries are of the same population size. The strategy could have positive results if the market for the additional sectors of the economy are sufficient large to increase employment and output of the economy, or if imports are curbed in favour to domestic production.

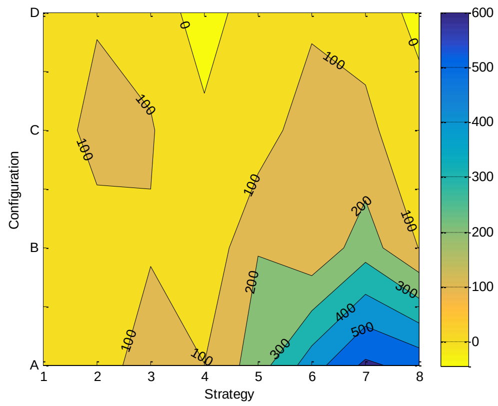
Figure 9

Comparison of catch-up strategies

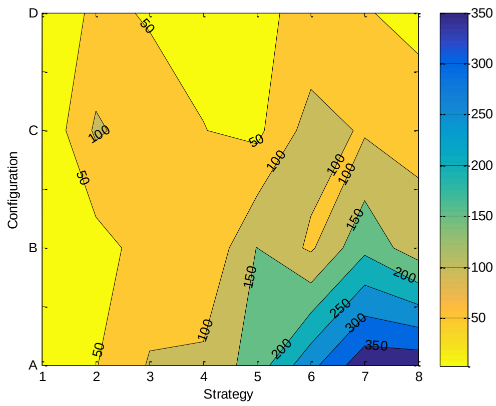
(A) Increase in diversification as compared with the global average



(B) Percentage increase in GDP



(C) Percentage increase in employment



Notes: Strategies:

1 - Focus on process innovation

2 - Focus on product innovation

3 - Benchmark

4 - Undervalued currency by 10%

5 - Focus on product emulation

6 - Target more complex products

7 - Focus on product emulation & Target more complex products

8 - Focus on product emulation & Target more complex products & undervalued currency by 10%

The next best strategy (number 5) is to focus exclusively on product emulation. This outperforms the benchmark scenario because now no R&D effort is lost in trying to find products that are new to the world in an adjacent possible that is limited by the reduced set of technologies available in the country. The strategy generates more innovation, a more diversified economy, higher output and employment. Focusing on emulation also outperforms the strategy of undervalued currency in terms of GDP and employment because more new products emerge, production and exports increase, and prices of imports are not negatively affected.

More effective than focusing on product emulation is the strategy of targeting emulation on new products that have above average complexity (strategy number 6). That strategy does not enforce a concentration of R&D on emulation (product and process innovation may as well coexist), but requires that whenever product emulation is pursued, such emulation target potential new products that are in the adjacent possible and that have product complexity above the country's average. Products with above average complexity are produced and exported in relatively fewer countries, hence the competition is lower and market shares and markups are higher. That increases not only output and employment but also makes available more resources for R&D and increases the innovation and the resulting diversification.

The combination of both strategies discussed above is even more effective (Strategy number 7): it dedicates all R&D effort to emulation and targets products with above average complexity. Now the positive effects of these strategies reinforce each other. More innovation is generated and the new products created face lower competition than the average product already produced in the country. Market shares and markups are higher; more resources can be dedicated to R&D, creating a virtuous cycle.

The addition of an undervalued currency (strategy 8) results in higher gains in diversification, but generally has a lower performance in terms of GDP and employment given the reasons already discussed above related to lower export prices and relatively higher prices of imports.

Therefore, in this simulation experiment, strategy 7 is the one that results in higher gains in most of the different configurations. The gains in following this strategy represented in increases in GDP by six- to eight-fold and increases in employment by three- to four-fold.

The main implication of these results is the realisation that economic development happens through the diversification of economies towards more complex products. Development policies should be designed and implemented aiming at facilitating that process. The implementation of such strategic diversification involves the selective promotion of new economic activities over traditional ones through the use of targeted industrial, infrastructure, trade, investment and private sector development policies. A key element in the implementation of those policies is the identification of the appropriate sectors and products to target based on country's productive structure and changes in global demand.

7 Conclusions

The model presented in this paper replicates empirical regularities that emerge from the international relations of countries and describes the channels through which diversification affects and is affected by structural economic dynamics.

The model directly contributes to the stream of literature based on Pasinetti's (1981, 1993) models by formalizing his framework of international trade. The model also contributes to the stream of the literature on structural dynamics that adopts Keynes-Kalecki principle of effective demand to determine the output of each sector, as opposed to models that assume that output is determined by full employment of labour and capital based on Say's law and Walras's law. The model also contributes to the study of the relationship between innovation and demand (Gualerzi, 2012) by connecting technologies to products, and products to human needs.

The model is used to study possible strategies for diversification of poorer countries, identifying those that could result in faster catch up. These results could help to better target action for the implementation of the internationally agreed goals. For example, the 2030 Agenda for Sustainable Development lists the target to "[A]chieve higher levels of economic productivity through diversification, technological upgrading and innovation, including through a focus on high-value added and labour-intensive sectors" (target 8.2). The results show that the strategy that results in higher increases in diversification is the one that combines a focus on emulation and that targeted products nearby in the product space with above average complexity.

The model could be used in numerous other analyses related to diversification and structural economic dynamics. For example, it could also study the relation between innovation and emulation and the effect of policies that promote or hinder technology transfer. The model could be used to study the economic dynamics of less populated and less diversified countries, such as the small island developing States, and how innovation and diversification in other countries could affect their possibilities for catch up. The model could also be extended to explore the relation of trade costs with diversification, a question of great importance for landlocked developing countries.

Another important extension of the model to make it more relevant to the study of the effects of diversification on sustainable development would be to consider environmental externalities created by the different sectors (e.g. carbon emissions, water pollution).Future research using the model is planned to address these questions.

References

Andersen, E.S. (2001). Satiation in an evolutionary model of structural economic dynamics. *Journal of Evolutionary Economics* (2001) 11:143-164.

Araujo, R. A. (2013). Cumulative causation in a structural economic dynamic approach to economic growth and uneven development. *Structural Change and Economic Dynamics* 24 (2013) 130-140.

Araujo, R. A. and Teixeira, J. R. (2004). A Pasinettian approach to international economic relations: the pure labor case. *Review of Political Economy*, Taylor & Francis Journals, vol. 16(1), pages 117-129.

Araujo, R. A. and Trigg, A.B. (2015). A neo-Kaldorian approach to structural economic dynamics. *Structural Change and Economic Dynamics* 33 (2015) 25-36.

Arthur, B. (2009). The nature of technology: what it is and how it evolves. Penguin.

Balland, P.A., Boschma, R.A. and Frenken, K. (2015). Proximity and innovation - From statics to dynamics. regional studies, 49 (6), (pp. 907-920).

Boschma, R.A. (2005). Proximity and innovation: A critical assessment. Regional Studies, Vol. 39.1, pp. 61–74, February 2005.

Bresser-Pereira, (2012). Structuralist macroeconomics and the new developmentalism. *Brazilian Journal of Political Economy*, 32(3,128),347-366.

Cadot. O., Carrere, C. and Strauss-Kahn, V. (2012). Export diversification: What's behind the hump? *The Review of Economics and Statistics*, 93(2), 590–605.

Clower, R.W. (1965), The Keynesian counter-revolution: a theoretical appraisal, The theory of interest rates, edited by F.H.Hahn and F. Brechling, International Economic Association Series, Macmillan, ch. 5, pp. 103-125.

De Paredes, J, Cárdenas, R, & Garcés, D 2013, 'Unit price information on the reference price formation', *Journal Of Product & Brand Management*, 22, 5/6, p. 413.

Dosi, G. (1982). Technological paradigms and technological trajectories. *Research Policy*, vol. 11, pp. 147-162.

________ (1988). Sources, procedures and microeconomic effects of innovation. *Journal of Economic Literature*, Vol. XXVI (September 1988), pp. 1120-1171.

Duchin, F. (2005). A World Trade Model based on comparative advantage with *m* regions, *n* goods, and *k* factors. *Economic Systems Research*, 17(2), 141-162.

ESCAP (2011). *Economic and Social Survey of Asia and the Pacific 2011: Sustaining Dynamism and Inclusive Development: Connectivity in the Region and Productive Capacity in Least Developed Countries*. Sales No. E.11.II.F2.

_______(2014). *Economic Diversification in Asian Landlocked Developing Countries: Prospects and Challenges*. Sales No. E.15.II.F.3.

_______(2015). *Asia-Pacific Countries with Special Needs Development Report 2015: Building Productive Capacities to Overcome Structural Challenges*. Sales No. E.15.II.F.9.

Freire, C. (2011). *Productive capacities in Asia and the Pacific*. ESCAP MPDD Working Papers WP/11/17, August 2011.

______(2013a). *Methodology to identify opportunities for development-oriented quality innovation: Application in the case of least developed countries*, in: Latif Al-Hakim and Chen Jin, eds, Quality Innovation: Knowledge, Theory, and Practices, IGI Global, USA.

______ (2013b). Strategies for structural transformation in South Asian countries, Seoul Journal of Economics 26, No. 3 2013: 311-336.

Freitas, M., Salvado, S. (2008). *Industry similarities, comparative advantage and upscale opportunities for Portugal in the process of structural transformation: an empirical assessment based on 2005 trade data*. GEE, September.

Freitas, M., Salvado, S., Nunes, L., Neves, R.C. (2013). *Productive experience and specialization opportunities for Portugal: an empirical assessment.* GEE papers Nr. 10, July 2013.

Gerschenkron, A. (1962). *Economic backwardness in historical perspective.* Harvard University Press, Cambridge, MA.

Gibson, B. (2010). *The structuralist growth model.* In Setterfield, M. (Ed.) (2010). Handbook of Alternative Theories of Economic Growth. Edward Elgar.

Grewal, D, Roggeveen, A, and Nordfält, J 2014, 'Pricing in the modern world: the role of price-related and non-price related cues', Journal of Product & Brand Management, September.

Grossman, G.M. and Helpman, E. (1991). Innovation and growth in the global economy (Cambridge MA: The MIT Press).

Gualerzi, D. (2012). *Growth theory, structural dynamics and the analysis of consumption.* In Arena, R. and Port, P.L (Eds.), 2012. Structural dynamics and economic growth. Cambridge University Press.

Gullstrand, J., Olofsdotter, K. and Thede, S. (2014). Markups and export-pricing strategies. Review of World Economics (2014) 150:221–239.

Hausmann, R., and Hidalgo, C. (2011). The network structure of economic output. *Journal of Economic Growth*, 16(4), 309-342.

Hausmann, R., and Klinger, B. (2008). *Achieving export-led growth in Colombia.* Center for International Development at Harvard University, CID Working Paper No. 182 September 2008.

Helpman, E., and Krugmann, P. R. (1985). Market structure and foreign trade. Cambridge, MA: MIT Press.

Hidalgo, C.A., Klinger, B., Barabasi, A-L, and Hausmann, R. (2007). The product space conditions the development of nations. *Science*, 317(5837), 482-487.

Hidalgo, C., and Hausmann, R. (2009). The building blocks of economic complexity. PNAS, 106(26), 10570-10575.

Imbs, J. and Wacziarg, R. (2003). Stages of diversification. *American Economic Review*, 93(1), 63-86.

Kauffman, S. A. (2008). *Reinventing the sacred.* Basic Books.

Klimek, P., Hausmann, R. and Thurner, S. (2012). Empirical confirmation of creative destruction from world trade data. CID Working Paper No. 238, April 2012.

Klinger, B, . and Lederman, D. (2004). *Discovery and development: An empirical exploration of 'New' Products.* World Bank Policy Research Working Paper 3450, November 2004.

Krugman, P. R. (1979). Increasing returns, monopolistic competition, and international trade. *Journal of International Economics*, 9 (1979), 469-479.

Lei, H. and Zhang, J. (2014). Capabilities's substitutability and the "S" curve of export diversity. EPL (Europhysics Letters) 105 (6), 68003.

Los, B. and Verspagen, B. (2006). The evolution of productivity gaps and specialization patterns. *Metroeconomica*, Wiley Blackwell, vol. 57(4), pages 464-493, November.

Pasinetti, L. (1993). *Structural economic dynamics: A theory of the economic consequences of human learning.* Cambridge University Press.

Ricardo, D. (1821). On the principles of political economy and taxation. 1821. Library of Economics and Liberty. 17 April 2016. <http://www.econlib.org/library/Ricardo/ricP.html>.

Rodrik, D. (2008). The real exchange rate and economic growth, *Brookings Papers on Economic Activity*, Economic Studies Program, The Brookings Institution, vol. 39(2 (Fall)), pages 365-439.
Romer, P.M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5), S71-S102.
Saracco, F., Di Clemente, R., Gabirelli, A., and Pietronero, L. (2015). From Innovation to diversification: A simple competitive Model. *PLOS ONE*, DOI: 10.1371/ journal.pone.0140420. November 6, 2015.
Saviotti, P. and Pyka, A. (2004a). Economic development by the creation of new sectors. *Journal of Evolutionary Economics,* 2004(14), 1–35.
______ (2004b). Economic development, qualitative change and employment creation. *Structural Change and Economic Dynamics*, 15(3), 265-287.
______ (2004c). Economic development, variety and employment. *Revue économique*, 55(6), 1023-1049.
Setterfield, M. (2010). *An introduction to alternative theories of economic growth.* Setterfield, M. (Ed.) (2010). Handbook of Alternative Theories of Economic Growth. Edward Elgar.
Smith, A. (1776). *An inquiry into the nature and causes of the wealth of nations.* (Reprinted by Oxford University Press, 2008).
Solow, R. M. (1956). A contribution to the theory of economic-growth. Quarterly Journal of Economics, 70(1), 65–94.
Solow, R. M. (1957). Technical change and the aggregate production function. Review of Economics and Statistics, 39(3), 312–320.
Thirlwall, A. P. (1979). The balance of payments constraint as an explanation of international growth rate differences. Banca Nazionale del Lavoro Quarterly Review, 128(1), 45-53.

Appendix A. Variables of the model

Exogenous	
N_k (*k=1,..,R*)	total population in country *k*
L_k (*k=1,..,R*)	total labour available for production sectors in country *k*
State variables	
$l_{j,k}$ (*j=1,..,m; k=1,..,R*)	labour coefficient (labour input per unit of output) to produce commodity *j* in country *k*
$c_{j,k}$ (*j=1,..,m; k=1,..,R*)	coefficient of consumption per capita of commodity *j* in country *k*
$MK_{j,h,k}$ (*j=1,..,m; h,k=1,..,R*)	markup added to the price of commodity *j* produced in country *k* and consumed in country *h*
w_k (*k=1,..,R*)	wage rate in country *k*
Endogenous	
$p_{j,h,k}$ (*j=1,..,m; h,k=1,..,R*)	price of commodity *j* produced in country *k* and consumed in country *h*
$c_{j,h,k}$ (*j=1,..,m; h,k=1,..,R*)	coefficient of consumption per capita in country *h* of commodity *j* produced in country *k*
$Q_{j,k}$ (*j=1,..,m; k=1,..,R*)	quantity of commodity *j* produced in country *k*
$E_{j,k}$ (*j=1,..,m; k=1,..,R*)	employment in sector *j* in country *k*
E_k (*k=1,..,R*)	total employment in country *k*
$Y_{j,k}$ (*j=1,..,m; k=1,..,R*)	output of sector *j* in country *k*
Y_k (*k=1,..,R*)	total output in country *k*
y_k (*k=1,..,R*)	output per capita in country *k*
$Exp_{j,k}$ (*j=1,..,m; k=1,..,R*)	household expenditure in sector *j* in country *k*
Exp_k (*k=1,..,R*)	total household expenditure in country *k*
exp_k (*k=1,..,R*)	per-capita household expenditure in country *k*

Appendix B. Sensitivity analysis

In this section, we run sensitivity tests to study how the macroeconomic results of the model are affected by the choices of parameters related to the arrival rate of innovation. The objective of this analysis is to show that changing the pace of product innovation and emulation does not change the results of the model related to the stylized facts. Technological change is the key driver of diversification and it is important to verify that changing the rate of innovation does not change the results of the model. The analysis in this section will focus on the stylized facts 1 and 2 related to the association of diversification with GDP and with the average ubiquity of exports.

To conduct those tests, we run simulations of the model 100 times considering a period of 50 time units to test different realizations of the stochastic process that uses the same set of initial parameters. For that analysis, we consider 10 countries initially trading 6 products. The countries have the same population size (100 people), labour and consumption coefficients at the beginning; therefore they have the same productivity, income and consumption levels. We track results related to diversification and output (GDP) for all countries, as well as inequality across countries.

The initial parameters are as follows:

- labour coefficient $l_1 = l_2 = \cdots = l_{10} = \{0.5, 0.5,0.5,0.5,0.5, 0.5\}$;
- coefficient of consumption per capita $c_1 = c_2 = \cdots = c_{10} = \{0.01, 0.01,0.01, 0.01,0.01, 0.01\}$;

- u = 0.4; 4 out of 10 potential new products are useful;
- wager rate in country 1 $w_1 = \$ 1$;
- $\beta = 2$; Saturation of consumption not higher than 100% above the initial consumption per capita.

For each set of simulations, we vary the parameters related to the arrival rate of product innovation ($\lambda_k^{product}$) and product emulation ($\lambda_k^{product\_emulation}$). For example, for a given value of the parameter of arrival rate of product innovation we make emulation be 'very easy', by considering the parameter of the rate of arrival of product emulation as ten times of the rate of arrival of product innovation, and then increase the level of difficulty in equal intervals until product emulation becomes as difficult as product innovation ($\lambda_k^{product\_emulation} = \lambda_k^{product}$). The rate of arrival of product innovation takes the values {1/100, 1/125, 3/500, 1/250, 1/500}. The scenario in which $\lambda_k^{product} = 1/100$ indicates that a researcher finds a new product on average once every 100 periods, while the scenario in which $\lambda_k^{product} = 1/500$ a new product is expected to be discovered by one researcher once every 500 periods. During the simulations, we consider that process innovation is as difficult as product innovation, and process emulation as difficult as product emulation:

- $\lambda_k^{process} = \lambda_k^{product}$
- $\lambda_k^{process\_emulation} = \lambda_k^{product\_emulation}$

The average of the results of the 100 runs for each set of parameters is presented in Table 3 in relation to the replication of the stylized fact 1. The table shows the slope of the relationship between the logarithm of diversification and the logarithm of GDP. The rows show the different arrival rates of product innovation, while the columns show the arrival rates of product emulation as a multiple of the arrival rate of product innovation. For example, the top right cell of the table shows the result of the simulations in which product innovation and emulation are faster, while the cell at the left and bottom of the table shows the result for simulations with slower innovation and emulation. The table shows that, in all combinations of rate of product innovation and emulation tested, diversification is positively associated with GDP, which replicates stylized fact 1. The slope increases when product innovation is slower (i.e. it increases from top to bottom). The reason is because a slower rate of product innovation implies fewer new products, and consequently fewer products that could be emulated, which reduces the chances for less diversified countries to catch up with the countries that have innovated first. As expected, the slope also increases when emulation becomes more difficult (i.e. from right to left).

Similarly, Table 4 shows the results related to the negative association between diversification and average ubiquity of exports (stylized fact 2). The table shows that all combinations of arrival rate of product innovation and emulation that were tested result in negative values for the slope of that association, which replicates the stylized fact. The table shows that the slope of the association becomes more negative when product innovation and emulation become slower (i.e. when one moves from top to bottom and from right to left in the table). Slower product innovation and emulation reduces the chances of less diversified countries to emulate the production of first movers and, as result, the latter benefits from lower competition and lower average ubiquity of exports, which is reflected in a steeper downward slope.

Table 3

Slope of the relationship between logarithm of diversification and logarithm of GDP (ln(k_{c0}) x ln(*GDP*))

Arrival rate of product innovation	Arrival rate of product emulation (multiple of the arrival rate of product innovation)									
	1x	2x	3x	4x	5x	6x	7x	8x	9x	10x
1/100	2.78	2.54	2.34	2.17	2.17	2.10	2.04	1.98	1.91	1.91
1/125	2.98	2.65	2.40	2.33	2.21	2.14	2.08	2.04	1.98	2.01
3/500	3.19	2.90	2.64	2.51	2.39	2.27	2.18	2.10	2.10	2.05
1/250	3.67	3.41	3.04	2.80	2.64	2.52	2.38	2.35	2.25	2.19
1/500	4.55	4.25	3.98	3.67	3.47	3.06	3.18	2.90	2.87	3.01

Table 4

Slope of the relationship between logarithm of diversification and logarithm of average ubiquity (ln(k_{c0}) x ln(k_{c1}))

Arrival rate of product innovation	Arrival rate of product emulation (percentage of product arrival rate of product innovation)									
	1x	2x	3x	4x	5x	6x	7x	8x	9x	10x
1/100	-0.7	-0.7	-0.6	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4	-0.4
1/125	-0.8	-0.7	-0.6	-0.6	-0.5	-0.5	-0.5	-0.5	-0.4	-0.4
3/500	-0.8	-0.7	-0.7	-0.6	-0.6	-0.6	-0.5	-0.5	-0.5	-0.5
1/250	-0.8	-0.8	-0.7	-0.7	-0.7	-0.6	-0.6	-0.6	-0.5	-0.5
1/500	-0.9	-0.8	-0.8	-0.8	-0.8	-0.7	-0.7	-0.7	-0.7	-0.7

In summary, changes in the rate of product innovation and emulation do not affect the results of the model in terms of replication of stylized facts.

Appendix C. Product complexity

We use the method of reflections proposed by Hidalgo and Hausmann (2009) to calculate the complexity of a product. The method uses a network of countries connected to the products that they export to build a matrix M_{cp} which elements are set to 1 if country c exports product p and 0 otherwise. The method calculates measures of diversification and ubiquity iteratively, using the following formulas:

$$k_{c,N} = \frac{1}{K_{c,0}} \sum_p M_{cp} k_{p,N-1} \qquad \text{(B.1)}$$

$$k_{p,N} = \frac{1}{K_{p,0}} \sum_c M_{cp} k_{c,N-1} \qquad \text{(B.2)}$$

Where $k_{c,0}$ is the number of products exported by country c and $k_{p,0}$ is the number of countries that export product p.

The measure of product complexity (*PCOMP*) is taken as the normalized value of the k_p value of the 5th iteration of the method of reflections:

$$PCOMP = \frac{k_{p,5} - \overline{k_{p,5}}}{\sigma} \qquad \text{(B.3)}$$

Where $\overline{k_{p,5}}$ is the mean and σ is the standard deviation of the distribution of $k_{p,5}$.